CYLINDER CONDENSATION IN UNSUPERHEATED STEAM ENGINES

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Introduction

The following analysis is intended as a starting point on the difficult topic of cylinder condensation. It is somewhat idealised, but represents scale effects and gives at least an order of magnitude value for the condensation rate during the admission stage of the steam engine cycle. As in so many other fields, John Perry[1] made a remarkable contribution to the topic about one hundred years ago; not only had he collected much experimental data, but in his usual style he set up theoretical models of several aspects of the process. One is tempted to say that all he lacked was a computer, but that might have denied him the fun of exercising his considerable skill in applied mathematics! He was aware of the strong effect of a liquid layer in determining condensation rate and of the effect the presence of a non-condensible gas might have in reducing it. He postulated that whilst condensation occurs during admission, the liquid layer probably evaporates during the later stages of expansion and during exhaust, and supported this idea with a formulation of the fluctuation in the surface temperature of the cylinder, and its attendant transient conduction process. He had a good idea of the thickness of the water layers likely to be formed - thin enough to remain as layers until they were evaporated - but he does not seem to have taken into account the controlling effect of the thermal resistance of the layer on condensation rate.

For the practical engineer at the end of the 19th century cylinder condensation - particularly in unsuperheated engines - was a major adverse effect on efficiency. It was defined in terms of the 'Missing Quantity' - a euphemism for the amount of steam consumed that could not be accounted for when the indicator card was analysed to yield the amount of steam that was actually present as vapour in the cylinder during expansion. It appears that the 'missing quantity' was in many cases as large as the mass of steam usefully employed. It is tempting to imagine that this was simply the result of heat loss from the outer surfaces of the cylinder, but Perry realised that this heat loss was far too small to account for such a condensation rate - hence the idea of transient conduction and condensation at the relatively cold cylinder wall during admission, followed by evaporation, and possibly even superheating of the cooler steam left in the cylinder during the exhaust stroke. Thus, whilst the net heat loss was small, the transient heat transfer process removed vapour by condensation at the start of the working stroke and replaced it by evaporation of the condensate layer towards the end - too late to produce any significant further work.

Since Perry's time a great deal has been learned about the process of condensation. The kinetic theory of gases enables one to calculate the rate at which gas or vapour molecules impact upon a solid or liquid surface. Equilibrium between steam and water is a dynamic situation involving 'condensation' of molecules that impact on the water surface, balanced by the 'evaporation' of molecules at an equal rate. In the non-equilibrium state, when the water surface is below saturation (i.e. equilibrium) temperature, the gross condensation rate remains high but the evaporation rate is diminished, resulting in a net condensation rate. On this basis the condensation coefficient (i.e. the net condensation rate divided by the temperature difference between the vapour and the surface of the water layer) would be very large indeed. For example, at atmospheric pressure a temperature difference of 1°C would result in a net condensation rate of about 3.4 kg/sq.m sec (1860 lb/ sq.ft hr), corresponding to a heat transfer rate of 0.76 kW/sq.cm ($1.8\ 10^6$ Btu/sq.ft hr). This is to be compared with the heat transfer rate through a 0.025 mm (0.001") thick water layer, with the same (1°C) temperature drop across it, of about 0.0027 kW/sq.cm (6400 Btu/sq.ft). Clearly the resistance of the water layer dominates, and this forms the basis of the calculations which follow.

Condensation is greatly reduced by superheating the steam supplied to an engine; in fact this effect is probably more important than the increase in thermodynamic efficiency resulting from superheat. The problem here is to determine the degree of superheat required to inhibit condensation. This will form the topic of a separate note (I hope!).

Outline of the Analysis

During one engine cycle the surfaces of the cylinder, ports and valve will be exposed to steam ranging in temperature from steam chest conditions to exhaust conditions. The temperature of the bulk of the cylinder will settle down to an intermediate value, and the transient conduction of heat into and out of the cylinder walls will alternately raise and lower the temperature of the surface above and below this value. The penetration of this temperature fluctuation will generally be quite small compared with the thickness of the cylinder wall; nevertheless it will be capable of extracting from the steam and later releasing to it a significant amount of heat. The processes of condensation into and evaporation from a water layer are similar, so I shall assume that the bulk of the cylinder assumes a temperature equal to the arithmetic mean of the steam chest and exhaust temperatures. With these assumptions the problem resolves into one of transient conduction into the cylinder through a water layer of varying thickness. Ideally these processes should be tracked through a complete engine cycle - a formidable problem! However it is possible to deal more simply with the admission phase of the cycle, and thus to form a reasonable estimate of the 'Missing Quantity'

Clearly, the resistance of the water layer as it builds up is the controlling factor in heat transfer to and from the cylinder wall. If we neglect wiredrawing, the saturation temperature of the steam during admission to the cylinder will remain constant, and with it the temperature of the surface of the water layer with which it is in contact. (This is a consequence of the very large condensation coefficient mentioned in the Introduction.) The rate of heat transfer through the water layer will certainly reduce as the layer increases in thickness and in addition it may change if the temperature of the cylinder surface changes. At first it appeared that the only way out would be a rather messy numerical solution of the partial differential equation governing transient conduction in the cylinder coupled with the varying boundary condition imposed by the water layer. However, serpendipity intervened, and a simple analytical solution was found!

It will be shown below that the interface between water layer and cylinder remains at a constant temperature during heat transfer from the steam to the cylinder. The value of this temperature is determined by the thermal properties of the material from which the cylinder is made, together with the thermal properties of the water layer. This arises from the happy accident that both processes (i.e. transient conduction in the cylinder, and conduction through a water layer of increasing thickness) give a heat transfer rate which varies inversely as the square root of time; thus the two processes when placed in series can share the same interface temperature. The resulting equations can be integrated to give the amount of steam condensed during the admission phase - i.e. the 'Missing Quantity'. A sketch of the form of temperature distribution at successive intervals of time (1< 2 < 3) is shown in Fig.1

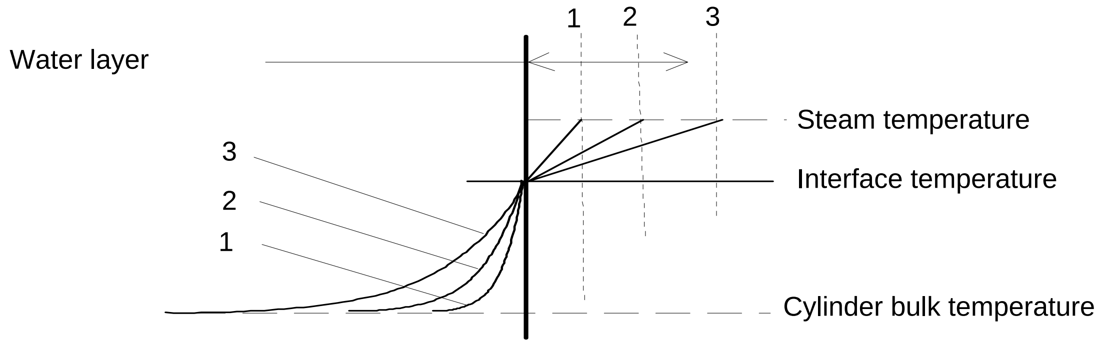


Fig.1 Successive temperature distributions

Conduction into the cylinder wall

In normal operation the surface temperature of the cylinder will fluctuate above and below some mean temperature. For the present purpose we assume that the cylinder is initially at a uniform temperature equal to the mean of the inlet and exhaust steam temperatures. Provided we check that the penetration of the temperature disturbance into the cylinder is small we may treat the cylinder as a semi-infinite solid, in which case the temperature distribution θ following a step increase of surface temperature of $\Delta\theta_s$ is given by:

$$\theta = \theta_0 + \Delta\theta_s \,\mathrm{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right) \quad . \quad . \quad . \qquad (1)$$

where x = distance into cylinder wall

κ = thermal diffusivity of cylinder (= $k_s/\rho_s c_s$)

k_s= thermal conductivity of cylinder material

ρ_s = density of cylinder material

c_s = specific heat of cylinder material

θ_0 = initial temperature of cylinder

t = time

[We can now check that the cylinder is thick enough to justify the 'semi-infinite' assumption. For an engine speed of 300 rpm the transient during admission of steam will last 0.1 second at most, and using this value together with a thermal diffusivity of 3.10^5 sq.m/sec (bronze) we find that the temperature disturbance has virtually decayed to zero in a distance of around 5 mm (0.2"). With cast iron this distance will be halved, and higher speeds will further reduce the penetration.]

The heat flux (i.e heat transfer per unit area and per unit time) into the cylinder wall corresponding to equation (1) will be:

$$q = -k_s \left({\partial\theta}/{\partial x} \right)_{x=o} = \frac{k_s \Delta\theta_s}{\sqrt{\pi\kappa t}} \quad . \quad . \quad . \qquad (2)$$

[Note that the heat flux is proportional to 1/ $\sqrt{t}$]

Heat transfer through the water layer

As with conduction into the cylinder wall, assume that the temperature difference driving heat through the water layer remains constant. [Both assumptions will be justified later]. The heat flux through the layer is :

$$q = {k_f \Delta\theta_f}/{\lambda} \qquad (3)$$

where k_f = thermal conductivity of water
λ = thickness of water layer
$\Delta\theta_f$ = temperature difference across water layer

As discussed in the Introduction, the outer surface of the water layer is virtually at the saturation temperature of the steam admitted to the cylinder. Condensation will take place at this surface at a rate determined by the removal of the latent heat of condensation by conduction through the water layer into the cylinder. The condensation rate is therefore q/h_{fg} kg/sq.m sec, or expressed as a rate of increase in the thickness of the layer, $q/(\rho_f h_{fg})$ m/sec, where h_{fg} is the enthalpy of evaporation (or condensation!). Using this we can write down a differential equation for the thickness of the water layer, λ :

$$\frac{d\lambda}{dt} = \left(\frac{k_f \Delta\theta_f}{\rho_f h_{fg}}\right) . 1/\lambda \qquad (4)$$

This can be integrated directly, and together with the boundary condition that $\lambda = 0$ at $t = 0$ gives the result:

$$\lambda = \sqrt{\left(\frac{2k_f \Delta\theta_f}{\rho_f h_{fg}}\right).t} \qquad (5)$$

Inserting this in equation (3) gives the heat flux through the layer as:

$$q = \sqrt{0.5 k_f \rho_f \Delta\theta_f h_{fg}} . /\sqrt{t} \qquad (6)$$

As in the case of conduction into the cylinder wall, conduction through the water layer is therefore proportional to $1/\sqrt{t}$

Heat Transferred to the Cylinder

The two processes, conduction into the cylinder wall (equation 3), and conduction through the water layer (equation 6) both yield a heat flux inversely proportional to the square root of time; the assumption that the temperature of the cylinder wall remains constant (as do $\Delta\theta_s$ and $\Delta\theta_f$) is therefore valid when the two processes are placed in series. The values of the two temperature drops can be determined by equating the heat fluxes defined by equations 3 & 6, and noting that that the overall temperature drop $\Delta\theta = \Delta\theta_s + \Delta\theta_f$ .This overall temperature drop is of course equal to the difference between the saturation temperature of the steam and the initial temperature of the cylinder wall. Equating the heat fluxes gives:

$$\frac{\Delta\theta_f}{(\Delta\theta_s)^2} = \frac{k_s\rho_s c_s}{k_f\rho_f h_{fg}}\cdot\frac{2}{\pi} \qquad . \quad . \quad . \quad . \quad . \qquad (7)$$

Substituting $\Delta\theta$ - $\Delta\theta_s$ for $\Delta\theta_f$ leads to the quadratic equation in $\Delta\theta_s$:

$$\gamma(\Delta\theta_s)^2 + \Delta\theta_s - \Delta\theta = 0$$

i.e.

$$\Delta\theta_s = \left(-1 + \sqrt{1 + 4\gamma\Delta\theta}\right) / 2\gamma \qquad . \quad . \quad . \quad . \qquad (8)$$

where $\gamma = \dfrac{k_s\rho_s c_s}{k_f\rho_f h_{fg}}\cdot\dfrac{2}{\pi}$

Substituting $\Delta\theta_s$ from equation 8 into equation 2 we get the heat flux, q:

$$q = \sqrt{\frac{k_s\rho_s c_s}{\pi}}\cdot\left(\frac{\sqrt{1+4\gamma\Delta\theta} - 1}{2\gamma}\right)\cdot\frac{1}{\sqrt{t}} = \frac{\xi}{\sqrt{t}} \qquad . \quad . \quad . \qquad (9)$$

Thus the net heat transfer on an elementary surface of area dA if exposed to steam for time τ is:

$$dQ = dA.\int_0^\tau \frac{\xi}{\sqrt{t}}\,dt = 2\xi\sqrt{\tau}\;dA \;. \qquad . \quad . \quad . \quad . \qquad (10)$$

Thus if we know the thermal properties of the material of the cylinder and of water, and also the difference between the steam temperature and the initial temperature of the cylinder we can calculate the amount of steam that would be condensed on an element of surface exposed to steam for a period of time τ. The practical situation is that only the ports, cylinder head and the end face of the piston are exposed at the start of the admission process. As far as the cylindrical surface is concerned, we need to allow for the progressive exposure of that surface; this is dealt with in the next section.

The 'Missing Quantity'

In order to calculate the condensation on the cylindrical surface of the cylinder we need to know the rate at which it is exposed to the steam, and also the point of cut-off. If ω is the angular velocity and s = piston stroke, then taking a datum of t = 0 when the piston is at the beginning of its stroke, the surface exposed during a time interval dt at t is given by:

$$dA = \pi\,D_p\,\frac{\omega}{2}\sin(\omega\,t)dt \qquad . \quad . \quad . \quad . \quad . \qquad (11)$$

where D_p is the piston diameter

In general, an element of surface reached by the piston at time t will be exposed to steam for a period τ, where τ = t - t_c and t_c is the time of cut-off. The amount of heat transferred to it is given by equation (10). Thus the total amount of heat tranferred to the cylindrical surface up to the time of cut-off is:

$$Q_c = \pi D_p s \xi \frac{1}{\sqrt{\omega}} \int_0^{\omega t_c} \sqrt{\omega t_c - \omega t}\, \sin(\omega t)\, d(\omega t)$$
$$= \pi D_p s \xi \frac{I(\omega t_c)}{\sqrt{\omega}} \qquad . \qquad . \qquad (12)$$

where s is the piston stroke.

Notice that the definite integral $I(\omega t_c)$ depends only upon the value of ωt_c, which is the crank angle at cut-off.

Turning now to the heat transfer to the end cover and the face of the piston, the 'exposure' time in this case will be the time to cut-off t_c, and the total amount of heat transferred will be:

$$Q_p = \pi D_p^2 \frac{\xi}{\sqrt{\omega}} . \sqrt{\omega t_c} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (13)$$

If we now divide the sum of equations 12 & 13 by the heat required to condense all the steam in the cylinder at the point of cut-off, we get the 'Missing Quantity' M, or the ratio of excess steam admitted to the steam that would be required if there were no condensation. Thus:

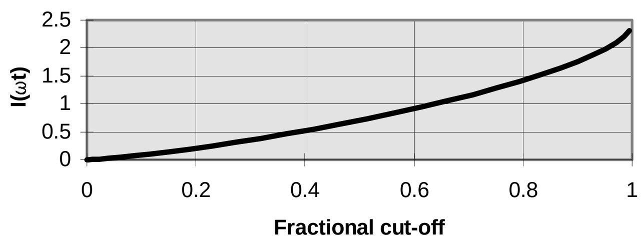
$$M = \frac{4\xi}{D_p \sqrt{\omega} f_c \rho_g h_{fg}} \left[I(\omega t_c) + \frac{D_p}{s} \sqrt{\omega t_c} \right] \qquad . \qquad . \qquad . \qquad (14)$$

where $f_c = \frac{(1 - \cos(\omega t_c))}{2}$ is the fractional cut-off, and ρ_g is the steam density

Evaluation of Equation 14

I am unable to integrate the definite integral $I(\omega t_c)$ analytically and have therefore integrated it numerically. Rather than presenting it as a function of crank angle at cut-off , ωt_c , it is more convenient to use the fractional cut-off, f_c , as independent variable. The result is shown in Fig.2

Fig. 2 Value of Integral I(ωt)



The following polynomial has been fitted to the curve in Fig.2

$$I(\omega t_c) = 4.243 f_c^4 - 6.826 f_c^3 + 4.529 f_c^2 + 0.327 f_c + 0.00584 \ldots \qquad (15)$$

The second term in the square brackets in Equation (14) would also be more conveniently expressed in terms of the fractional cut-off, and in this form becomes:

$$\left(D_p/s\right)\sqrt{\cos^{-1}\left(1-2f_c\right)}$$

The value of *M* can be calculated from the above equations. (The calculation is greatly facilitated by the use of a small computer programme, which can be supplied by the author)

Presentation of Results.

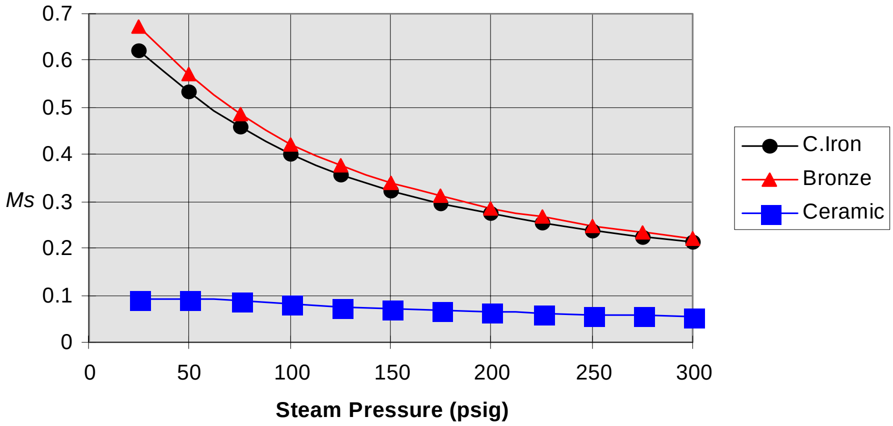
The traditional method of presenting equation (14) would be to cast it in terms of dimensionless groups; however, this would lose sight of the primary variables such as speed, steam pressure etc. which are the factors that readily come to mind when dealing with steam engines. So a rather less elegant but perhaps more useful method will be adopted. This involves setting up a 'standard engine' of known size and speed, operating at a known cut-off , and calculating the value of the missing quantity *Ms* for a range of steam chest pressures, and for several cylinder materials.

As the standard, we will take an engine with the following parameters:

Diameter of cylinder	2 in.
Stroke	4 in
Fractional cut-off	0.5
Speed	500 rpm

The result of this calculation is shown in Fig.3 for steam chest pressures from 25 psig to 300 psig and for bronze, cast iron and ceramic cylinders. (The ceramic chosen was porcelain - the only ceramic material for which I could readily obtain data!) Since all steam engines do not have cylinders 2″ diameter and run at 500 rpm etc. some means must now be found of scaling factors for size, speed, cut-off and D_p/s !

Fig.3 The Missing Quantity *Ms* for the Standard Engine (standard size, speed, c/o, D/s)



When calculating *Ms* for Fig.3 the thermal properties of the steam must be known, and also the difference between the steam temperature and the mean cylinder temperature. All these can be related to the steam pressure, which is the independent variable in Fig.3 I have assumed that since the cylinder is alternately exposed to steam at the boiler pressure, and steam at atmospheric pressure, its mean temperature will be *(boiler temp - 100)/2.*

The Effect of speed

The speed appears only in the $1/\sqrt{\omega}$ term in Equation (14), so the effect is for the Missing Quantity, to be reduced in proportion to the square root of the speed. This appears to be in line with Perry's estimates, and indeed with my own experiments. Since rotational speed in RPM is proportional to angular velocity, we may express the speed factor as

$$R = 1\Big/\sqrt{speed\ (rpm)/500(rpm)} \qquad . \qquad .(16)$$

The Effect of Size

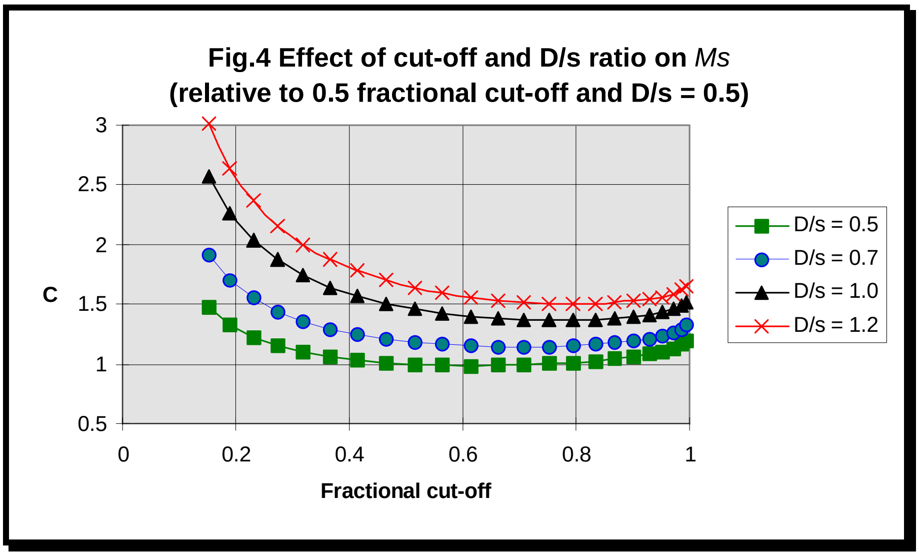
For a fixed ratio of $D_p\,/\,s$ size enters into Equation (14) only in the term $1/D_p$, so that the Missing Quantity increases inversely as the size of the engine. The size factor *S* is therefore:

$$S = 1\Big/\left(piston\ dia.\ in/2\ in\right) \quad . \quad . \quad . \quad . \qquad (17)$$

The effect of Cut-off and Piston dia./Stroke ratio

The effect of cut-off on *M* is governed by the terms $1/f_c$ and $\left[\ I(\omega\, t_c) + \frac{D_p}{s}\sqrt{\omega\, t_c}\ \right]$. The term $\omega\, t_c$ can be expressed in terms of the fractional cut-off, but the effect of this and D_p/s cannot be separated . The cut-off effect *C* (relative to the value for the 'standard engine' values of 0.5 fractional cut-off and D_p/s = 0.5) is shown in Fig.4 as a function of fractional cut-off for various values of D_p/s

$$C = \text{fn}\Big(\big(\ fractional\ cut-off\ /0.5\big), \big(D_p/s\big)\Big) \qquad . \qquad . \qquad (18)$$



The Value of the Missing Quantity for any engine and operating condition

Putting together the above results, we are able to scale the value of the Missing Quantity for the 'Standard Engine', *Ms*, so that it represents any set of data. Thus:

$$M = Ms \times R \times S \times C \qquad . \quad . \quad . \quad . \quad . \qquad (19)$$

where *Ms* is read from Fig.3, *C* from Fig.4, and *R* and *S* are obtained from Equations (16) and (17), respectively.

This way of expressing the results displays clearly the separate effects of speed, size, cut-off, D_p/s ratio, steam pressure, and cylinder material.

Illustrative Examples

The following results, all for cast iron cylinders, bring out the important trends.

D_p (in.)	*s* (in.)	Speed (rpm)	Pressure (psig)	Cut-off (fractional)	M	Perry (see below)
2	4	500	100	0.5	0.400	(0.71)
2	2	500	100	0.5	0.589	(0.71)
2	4	1000	100	0.5	0.284	(0.50)
2	4	500	200	0.5	0.275	
2	4	500	100	0.25	0.474	(1.20)
15	30	200	150	0.25	0.081	
15	30	200	150	0.5	0.068	(0.15)
15	30	50	150	0.5	0.137	(0.30)
1	2	500	75	0.7	0.895	
1	1	1000	100	0.3	1.015	(1.44)

(If you need more, ask me to send you the computer programme!)

John Perry ("The Steam Engine….", 1909) presents his students with the following rule:

$$\text{Missing Quantity} = 15\,(1 + r)/\,(d\sqrt{n})$$

where r is the expansion ratio, d is the cylinder diameter in inches, and n is the number of **strokes** per minute. The effect of pressure is not included, but from the context it seems likely that he had in mind pressures less than 100 psig. I have included a few values from Perry's rule in the above table. The trends are reasonably in agreement, but his figures are higher. However, Perry goes on to say, "Instead of 15 we might have as small a number as 5 in a well-jacketed, well drained cylinder of good construction with four double beat valves, and we might have as great a number as 30 or even more in badly drained and unjacketed engines with slide valves. This range obviously embraces the calculated values presented here!

The point of real interest is that all the trends exhibited by Perry's rule agree very closely with the theoretical results. Perry gives a pressure effect for condensing engines, for which *M* is inversely proportional to the square root of pressure - again a trend that is consistent with the theory.